

Question Paper

Quantitative Methods – I (131) : July 2003

Section A : Basic Concepts (40 Points)

- This section consists of questions with serial number 1 - 40.
- Answer all questions.
- Each question carries one point.

1. The set of natural numbers
 - a. Is a finite set
 - b. Starts with the number zero
 - c. Does not include zero
 - d. Includes only fractions
 - e. Includes only negative numbers.
2. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ is known as the
 - a. Commutative property of multiplication
 - b. Identity property of multiplication
 - c. Inverse property of multiplication
 - d. Associative property of multiplication
 - e. Cancellation property of multiplication.
3. Which property states that the addition of zero to a real number returns back the number itself?
 - a. Cancellation property of addition
 - b. Commutative property of addition
 - c. Associative property of addition
 - d. Identity property of addition
 - e. Inverse property of addition.
4. The least common multiple of a group of numbers is
 - a. The smallest number in the group
 - b. A multiple of only the smallest number in the group
 - c. A multiple of only the largest number in the group
 - d. The smallest multiple of only the smallest number in the group
 - e. The smallest number that can be divided by each number in a group of numbers without leaving a remainder.
5. The set of rational numbers
 - a. Includes positive numbers only
 - b. Includes negative numbers only
 - c. Includes those numbers which are of the form $\frac{p}{q}$, where p and q are integers, and $q \neq 0$
 - d. Includes those numbers which cannot be expressed in the form $\frac{p}{q}$, where p and q are integers
 - e. Includes negative integers only.
6. Which of the following is **false**?
 - a. If $b - a > 0$, then $a < b$
 - b. If $a < b$ and $b < c$, then $a < c$
 - c. If $a < b$, then $(a + c) < (b + c)$
 - d. If $a < b$ and $x < 0$, then $ax < bx$
 - e. If $a < b$ and $x = 0$, then $ax = bx$.

7. Which of the following intervals, defined by the end points **a** and **b**, includes those numbers which are greater than **a** and, less than or equal to **b**?
- (a, b)
 - (a, b]
 - [a, b)
 - [a, b]
 - None of the above.
8. A series of numbers is said to be in a geometrical progression if
- The successive numbers in the series differ by a constant value
 - The numbers in the series are all equal
 - The successive numbers in the series increase or decrease by a common ratio
 - The numbers in the series assume positive and negative signs alternately
 - The numbers in the series are all multiples of 10.
9. A quadratic equation is of the form $ax^2 + bx + c = 0$, where **a**, **b** and **c** are constants. If **c** is equal to zero, then the roots of the equation are
- Only positive values
 - Only negative values
 - All equal to zero
 - $\frac{b}{a}$ or $-\frac{b}{a}$
 - 0 or $-\frac{b}{a}$
10. The reciprocals of the terms in a harmonic progression are
- Always greater than 1
 - Always less than 1
 - Always negative values
 - In arithmetic progression
 - In geometric progression.
11. If **A**, **G** and **H** are the arithmetic mean, geometric mean and harmonic mean between two given quantities, then
- G** is the geometric mean between **A** and **H**
 - A** is the arithmetic mean between **G** and **H**
 - H** is the harmonic mean between **A** and **G**
 - A** = **G** always
 - A** = **H** always.
12. If three quantities viz, **a**, **b** and **c** are in harmonic progression, then
- $c = \frac{2ab}{a+b}$
 - $b = \frac{2ac}{a+c}$
 - $a = \frac{2bc}{b+c}$
 - $b = \frac{ac}{a+c}$
 - $c = \frac{ab}{a+b}$

13. The sum of an arithmetic progression (A.P) consisting of n terms is zero. Which of the following is **true**? (t_1 represents the first term and t_n represents the n th term of the A.P.)
- $t_1 = -t_n$
 - $t_1 = t_n$
 - $t_1 / t_n = 0$
 - $t_1 / t_n > 1$
 - $t_n / t_1 > 1$.
14. ${}^nC_r =$
- nC_n
 - nC_1
 - nP_r
 - ${}^nC_{n-r}$
 - n .
15. $\log_b a \times \log_a b =$
- 0
 - a
 - b
 - $a \times b$
 - 1.
16. $\log_a (M^n) =$
- M^n
 - $\log_a n$
 - $\log_a M$
 - $n \log_a M$
 - $\log_a (nM)$.
17. $\log_a M + \log_a N =$
- $\log_a (M + N)$
 - a
 - $\log_a (M \times N)$
 - 1
 - $M \times N$.
18. For any real number, the interval $(-\infty, a)$ represents the
- Set of all real numbers greater than a
 - Set of all real numbers greater than or equal to a
 - Set of all negative numbers less than a
 - Set of all negative numbers greater than a
 - Set of all real numbers less than a .
19. The domain of a function, $y = f(x)$, includes
- The values which may be assumed by y
 - The values of x for which the function is defined
 - The values of x for which the function is undefined
 - The values of x which are greater than a pre-specified value
 - The values of x which are less than a pre-specified value.

20. Which of the following statements is **true**?
- To every number in the domain, a function always associates a negative value
 - To all the numbers in the domain, a function does not associate any value in the range
 - To every number in the domain, a function always associates the same number in the range
 - To every number in the domain, a function always associates a unique number in the range
 - None of the above.
21. If the graph of a function involving one independent variable is a straight line, then the function is a
- Logarithmic function
 - Exponential function
 - Inverse function
 - Linear function
 - Non-linear function.
22. Which of the following statements is **true** with regard to a linear programming problem?
- The objective function is a statement of a constraint
 - The coefficients of the decision variables in the constraints represent the per unit contribution of the decision variables to the value of the objective function
 - The decision variables can not assume zero values
 - The constraint inequations specify the consumption of resources and the amount of resources available
 - The coefficients of the decision variables in the objective function represent the per unit consumption of resources by the decision variables.
23. Which of the following is **not** an assumption underlying linear programming?
- The objective to be accomplished can be expressed as a linear function of the decision variables
 - The constraints on the use of resources can be expressed as linear equations or inequations
 - The amount of resources consumed by each unit of the decision variables is uncertain
 - The decision variables can take non-negative values only
 - The total consumption of a resource is the sum of the resources consumed by each decision variable.
24. For a function, $f(x)$, $f'(x) = 0$, for all values of x . It can be said that $f(x)$
- Increases as x increases
 - Decreases as x increases
 - Has a constant value for all values in the domain of x
 - Increases at an increasing rate
 - Decreases at a decreasing rate.
25. For a function, $f(x)$, $f'(x) = 0$ for $x = a$. Which of the following is **correct**?
- $f(x)$ has a relative maxima at $x = a$
 - $f(x)$ has a relative minima at $x = a$
 - $f(x)$ will have a relative minima if $f''(x) < 0$ at $x = a$
 - $f(x)$ will have a relative maxima if $f''(x) < 0$ at $x = a$
 - None of the above.

26. If every item in a data set is increased by a constant **C**, then the arithmetic mean of the resulting data set will be equal to
- The mean of the original data set
 - C** + Mean of the original data set
 - C** – Mean of the original data set
 - Mean of the original data set $\div$ **C**
 - Mean of the original data set $\times$ **C**.
27. The mean of the squared deviations of all the observations in a data set is equal to the
- Standard deviation of the data set
 - Median of the data set
 - Mode of the data set
 - Range of the data set
 - Variance of the data set.
28. The events **B**, **C** and **D** are mutually exclusive and collectively exhaustive; **A** is another event which can jointly occur with **B**, **C** or **D**.
- $P(A \text{ and } B) + P(A \text{ and } C) + P(A \text{ and } D) =$
- 1.00
 - $P(D)$
 - $P(C)$
 - $P(B)$
 - $P(A)$.
29. Which of the following statements is **true** with regard to the exponential function $y = m \cdot a^x$?
- The exponential curve falls from left to right as the values increase along the **X**-axis if **$m > 0$** and **$0 < a < 1$**
 - The exponential curve falls from left to right as the values increase along the **X**-axis if **$m > 0$** and **$a > 1$**
 - The exponential curve is parallel to the **X**-axis if **$m > 0$** and **$0 < a < 1$**
 - The exponential curve is parallel to the **X**-axis if **$m > 0$** and **$a > 1$**
 - None of the above.
30. Which of the following indicates that the distribution of the data is negatively skewed?
- Arithmetic Mean $<$ Median $<$ Mode
 - Median $<$ Mode $<$ Arithmetic Mean
 - Mode $<$ Arithmetic Mean $<$ Median
 - Mode $<$ Median $<$ Arithmetic Mean
 - Median $<$ Arithmetic Mean $<$ Mode.
31. Which of the following statements is **false** with regard to the derivative of a function?
- It indicates the rate of change of the function at a fixed point
 - The slope of the tangent to a function at a point is equal to the derivative of the function at the point
 - The derivative may be said to be a function of the independent variable
 - The value of the derivative is always independent of the point where the derivative is evaluated
 - None of the above.
32. The totality of all possible outcomes of an experiment is called
- Sample space
 - Population
 - Event
 - Data
 - Sample.

33. Which of the following is **not** the correct step for arranging data in the form of class intervals?
- The lowest value should be included in the first class and the highest value should be included in the last class
 - The adjacent classes should not have any interval in between
 - The adjacent classes should overlap
 - Every item of data should be included in one and only one class
 - None of the above.
34. The average deviation of all items in the data from zero is equal to the
- Arithmetic mean
 - Median
 - Mode
 - Standard deviation
 - Variance.
35. Baye's theorem helps the statistician to calculate
- Subjective probability
 - Classical probability
 - Revised probability
 - Central tendency
 - Dispersion.
36. On the basis of which of the following, is an ogive drawn?
- Absolute frequencies
 - Relative frequencies
 - Cumulative frequencies
 - Probabilities
 - None of the above.
37. The sum of squared deviations of the items from the _____ is less than the sum of the squared deviations of the items from any other value.
- Harmonic mean
 - Arithmetic mean
 - Geometric mean
 - Median
 - Mode.
38. Which of the following is a relative measure of dispersion?
- Variance
 - Standard deviation
 - Range
 - Coefficient of variation
 - None of the above.
39. If every item in a data set is decreased by the same quantity then the standard deviation of the resulting data set
- Remains the same
 - Increases by the same quantity by which every data item is increased
 - Decreases by the same quantity by which every data item is increased
 - Increases by the square root of the same quantity by which every data item is increased
 - Decreases by the square root of the same quantity by which every data item is increased.

40. The appropriate mean for a set of ratios using the denominators of the ratios as weights is
- Geometric mean
 - Simple harmonic mean
 - Weighted arithmetic mean
 - Median
 - Mode.

END OF PART A

Section B : Problems (60 Points)

- This section consists of questions with serial number 41 - 70.
- Answer all questions.
- Points are indicated against each question.

41. Which of the following is **correct**?

- $g(t) = \frac{t+5}{t^2+t-12}$ is defined at $t = 3$
- $g(t) = \frac{t+4}{t^2-3t-10}$ is defined at $t = -2$
- $g(t) = \frac{t-2}{t^2-9t+20}$ is not defined at $t = 3$
- $g(t) = \frac{t-1}{t^2-5t+6}$ is not defined at $t = 2$
- $g(t) = \frac{t-5}{t^2-6t+8}$ is not defined at $t = 2$ but is defined at $t = 4$.

(1 point)

42. Which of the following is **correct**?

- $f(x)$ is a linear function and, $f(2) = 7$ and $f(7) = 32$. The slope of the function is -3
- $g(x)$ is a linear function and, $g(-5) = 40$ and $g(2) = -9$. $\therefore g(0) = 7$
- y is a linear function of x such that $\frac{y}{5} - \frac{x}{2} = 5$. The intercept of the function on the Y axis is 25
- y is a linear function of x . If $x = 2$, then $y = 20$. If $x = 5$, then $y = 44$. $\therefore$ If $x = 8$, $y = 60$
- None of the above.

(1 point)

43. Which of the following is **correct**?

- $g(x) = x^2 - 8x + 15$ If $x = -3$, then $g(x) = 0$
- $g(x) = x^2 + 2x - 8$ If $x = 4$, then $g(x) = 0$
- $g(x) = x^2 + x - 6$ If $x = -3$, then $g(x) = 0$
- $g(x) = x^2 - x - 6$ If $x = 2$, then $g(x) = 0$
- $g(x) = x^2 - 9x + 20$ If $x = 5$, then $g(x) \neq 0$.

(1 point)

44. Which of the following is/are **correct**?

- a. $f(t) = \frac{t^2 - 5t + 6}{t^2 - 9t + 18} \quad \therefore \lim_{t \rightarrow 3} f(t) = \frac{1}{6}$
- b. $f(t) = \frac{t^3 - 1}{t^2 - 1} \quad \therefore \lim_{t \rightarrow 1} f(t) = \frac{3}{2}$
- c. $f(t) = \frac{\sqrt{1+t} - \sqrt{1-t}}{t} \quad \therefore \lim_{t \rightarrow 0} f(t) = 0$
- d. $f(t) = \frac{t+1}{t+2} \quad \therefore \lim_{t \rightarrow \infty} f(t) = 0$
- e. All of the above.

(2 points)

45. Which of the following is/are **correct**?

- a. $f(t) = \frac{\ln t}{t} \quad \therefore f'(t) = \frac{1 - \ln t}{t^2}$
- b. $f(t) = (\ln t)^5 \quad \therefore f'(t) = \frac{1}{t} (\ln t)^2$
- c. $f(t) = (2t + 5)^5 \quad \therefore f'(t) = 5(2t + 5)^4$
- d. $f(t) = e^{\sqrt{t}} \quad \therefore f'(t) = \frac{e^{\sqrt{t}}}{2}$
- e. All of the above.

(2 points)

46. $f(t) = 112t - \frac{28}{15}t^2 - 600$

$f(t)$ is maximum at $t =$

- a. 0
- b. 15
- c. 30
- d. 112
- e. 600.

(2 points)

47. $f(x) = 15x^2 - 60x$

$f(x)$ is minimum at $x =$

- a. 2
- b. 5
- c. 15
- d. 20
- e. 60.

(2 points)

48. $f(x) = 3x^2 - (k + 12)x + 200$ Where k is a constant

If $f'(4) = 0$, then $k =$

- a. 4
- b. 6
- c. 10
- d. 12
- e. 24.

(1 points)

49. The total cost of production in a day, for a firm, is given by the following equation:

$$T(x) = 10500x - 20x^2 + \frac{1}{5}x^3$$

Where $T(x)$ = Total cost of production in a day
 x = Number of units produced in a day

How many units of the product should be produced in a day so that the average cost of the product is minimum?

- a. 5
- b. 10
- c. 20
- d. 50
- e. 100.

(2 points)

50. The mean age of a group consisting of both men and women was found to be 36 years. The mean age of the men in the group was found to be 40 years while the mean age of the women in the group was found to be 30 years.

Which of the following statements is/are **correct**?

- I. The number of men in the group can be found out from the given data.
- II. The number of women in the group can be found out from the given data.
- III. The total number of men and women in the group can be found out from the given data.
- IV. The percentage of men in the group is 60%.
- V. The percentage of men in the group is 40%.
- VI. The percentage of women in the group is 40%.
- VII. The percentage of women in the group is 60%.

- a. Only (I)
- b. Only (II)
- c. Only (III)
- d. Both (IV) and (VI)
- e. Both (V) and (VII).

(2 points)

51. The market value of an investment has undergone the following growth rates in different years:

Year	1	2	3	4	5	6	7
Growth rate (%)	5	8	-10	-4	6	10	7

Which of the following statements is/are **correct**?

- I. The appropriate average that should be applied for calculating the average growth rate over the period of seven years is arithmetic mean.
- II. The appropriate average that should be applied for calculating the average growth rate over the period of seven years is the harmonic mean.
- III. The appropriate average that should be applied for calculating the average growth rate over the period of seven years is the geometric mean.
- IV. The average growth rate over the seven year period is 3.14%.
- V. The average growth rate over the seven year period is 18.2%.
- VI. The average growth rate over the seven year period is 2.91%.

- a. Only (I)
- b. Only (II)
- c. Both (I) and (IV)
- d. Both (II) and (V)
- e. Both (III) and (VI).

(2 points)

52. As study was conducted on the daily wages earned by the unskilled labourers in an industrial area. The observations from the survey are given below:

Daily wages (in Rs.)	Number of labourers
25 – 35	15
35 – 45	25
45 – 55	32
55 – 65	28
65 – 75	18
75 – 85	12

The median daily wage earned by the unskilled labourers is

- 35.50
- 45.25
- 52.66
- 60.56
- 65.25.

(2 points)

53. The following details are available with regard to a sample of eight observations:

$$\sum_{i=1}^4 x_i = 64 \quad \sum_{i=2}^5 x_i = 80 \quad \sum_{i=5}^8 x_i = 66$$

$$\sum_{i=2}^4 x_i = 54 \quad \sum_{i=1}^8 x_i^2 = 2412 \quad x_5 = 26$$

The sample standard deviation is

- 6.54
- 16.25
- 17.36
- 42.77
- 130.

(2 points)

54. What is the harmonic mean of the following quantities?

20, 15, 30, 22, 12, 25, 18, 16

- 0.055
- 0.437
- 18.313
- 19.02
- 19.75.

(1 point)

55. There are three sets of data. The first set contains four quantities and their geometric mean is 1.252. The second set contains eight quantities and their geometric mean is 1.164. The third set contains five quantities and their geometric mean is 1.206. Each of the three sets of data is expanded by including one more quantity as follows:

The quantity 1.20 is included into the first set, the quantity 1.18 is included into the second set and the quantity 1.24 is included into the third set. What is the combined geometric mean of all the three sets of data after the expansion?

- a. 1.166
- b. 1.212
- c. 1.241
- d. 1.198
- e. 1.389.

(2 points)

56. A data set includes some quantities. The sum of reciprocals of the quantities in the data set is $\frac{7}{8}$. The harmonic mean of the data set is $3\frac{3}{7}$. The data set is expanded by including the quantities 5 and 10 into it.

What is the harmonic mean of the expanded data set?

- a. $\frac{7}{24}$
- b. $10\frac{2}{21}$
- c. $6\frac{2}{3}$
- d. $7\frac{3}{7}$
- e. $4\frac{12}{47}$

(2 points)

57. The mean and standard deviation of a sample consisting of 50 observations were worked out to be 40 and 22 respectively. Subsequently it was found out that there was a mistake in the process and one of the observations included in the sample was 150 instead of 50.

Which of the following statements is/are **correct**?

- I. The correct mean is 41.
- II. The correct mean is 38.
- III. The correct standard deviation is 23.35.
- IV. The correct standard deviation is 15.33.

- a. Only (I)
- b. Only (III)
- c. Only (I) and (III)
- d. Only (II) and (III)
- e. Both (II) and (IV).

(2 points)

58. A small contractor undertakes various types of jobs in the construction business. From his past experience he believes that the probability that he will get a plumbing contract is $\frac{2}{5}$ and the probability that he will not get a wiring contract is $\frac{1}{4}$. He also believes that the probability that he will get at least one of the two types of contract is $\frac{17}{20}$. The two types of contracts are independent of each other.

What is the probability that he will get both types of contract?

- a. 60%
- b. 75%
- c. 30%
- d. 70%
- e. 85%.

(1 point)

59. Five students viz, **P, Q, R, S** and **T** are independently trying to solve a problem in mathematics. The probabilities that they will be able to solve the problem are $\frac{2}{5}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{1}{2}$ and $\frac{3}{8}$ respectively.

What is the probability that the problem will be solved?

- a. 5.2%
- b. 0.83%
- c. 25.5%
- d. 56.7%
- e. 94.8%.

(2 points)

60. A bin in a hardware store, contains 125 bolts and 200 nuts. One-fifth of the bolts and three-fourth of the nuts are defective. One item is picked randomly from the bin.

What is the probability that the item is either a defective item or it is a nut?

- a. 92.3%
- b. 84.6%
- c. 7.7%
- d. 53.8%
- e. 38.5%.

(2 points)

- 61.** In a factory a particular part is manufactured by either of two machines, **A** and **B**. The quality control department in the factory has observed that the parts that are manufactured by either of the two machines can have either 0, 1, or 2 defects. The following observations have been made over a sufficiently long period of time:

- The probability that the part will be manufactured by machine **A** and it will have no defect is 42%.
- The probability that the part will be manufactured by machine **A** and it will have 1 defect is 12%.
- The probability that the part will be manufactured by machine **A** and it will have 2 defects is 6%.
- The probability that the part will be manufactured by machine **B** and it will have no defect is 24%.
- The probability that the part will be manufactured by machine **B** and it will have 1 defect is 10%.
- The probability that the part will be manufactured by machine **B** and it will have 2 defects is 6%.

Which of the following statements are **correct**?

- I. The probability that the part will be manufactured by machine **B** is 60%.
- II. The probability that the part will be manufactured by machine **A** is 60%.
- III. The probability that the part will have 1 defect is 12%.
- IV. The probability that the part will have 2 defects is 22%.
- V. The probability that the part will be free from defect is 66%.
- VI. If the part is manufactured by machine **B** then the probability that it will have no defect is 24%.
- VII. If the part is manufactured by machine **B** then the probability that it will have 1 defect is 10%.
- VIII. If the part is manufactured by machine **B** then the probability that it will have 2 defects is 15%.
- IX. If the part is manufactured by machine **A** then the probability that it will have 2 defects is 6%.
- X. If the part is manufactured by machine **A** then the probability that it will have 1 defect is 12%.
- XI. If the part is manufactured by machine **A** then the probability that it will be free from defect is 70%.

- a. (I) and (V) only
- b. (VI) and (X) only
- c. (II), (IV), (VI) and (IX)
- d. (I), (III), (VII) and (X)
- e. (II), (V), (VIII) and (XI).

(4 points)

- 62.** Two persons, **A** and **B**, committed a crime together. The police has arrested both of them for the purpose of interrogation. **A** tells truth 4 out of 5 times, while **B** tells truth 3 out of 4 times. The police will be interrogating them separately.

What is the probability that they will give contradictory statements during the course of interrogation?

- a. 80%
- b. 75%
- c. 15%
- d. 35%
- e. 20%.

(1 point)

63. An NGO has been studying the literacy level among the male and female adults, in a certain village. 3 out of every 5 adults in the village are males. It is observed that 1 out of every 2 adult males, is illiterate and, 4 out of every 5 adult females are illiterates.

If an adult is randomly selected and found to be illiterate then, what is the probability that the adult is a female?

- a. 32%
- b. 25.8%
- c. 51.6%
- d. 12.9%
- e. 62%.

(3 points)

64. In an arithmetical progression (A.P) j times the j^{th} term is equal to k times the k^{th} term. The first term is a and common difference is d . What is the $(j + k)^{\text{th}}$ term of the A.P.?

- a. 0
- b. $(j + k) d$
- c. $(j + k - 1) d$
- d. a
- e. d .

(2 points)

65. In an arithmetical progression (A.P.) the first term is 4 and the last term is 24. The sum of all the terms in the A.P. is 154. What is the common difference of the A.P.?

- a. 10
- b. 6
- c. 4
- d. 2
- e. 11.

(2 points)

66. Three numbers are in arithmetical progression. The sum of the three numbers is 18 and the sum of their squares is 140.

Which of the following statements are **correct**?

- I. The first number is 6 or 10.
- II. The first number is 2 or 10.
- III. The first number is 6.
- IV. The second number is 6.
- V. The second number is 2 or 6.
- VI. The second number is 6 or 10.
- VII. The third number is 6 or 2.
- VIII. The third number is 6.
- IX. The third number is 10 or 2.

- a. (I) and (VI)
- b. (V) and (VII)
- c. (II) and (VIII)
- d. (III) and (IX)
- e. (II), (IV) and (IX).

(2 points)

- 67.** Three numbers are in geometrical progression such that their sum is 21 and their product is 216.

Which of the following statements are **correct**?

- I. The first number is 6.
 - II. The first number is 6 or 12.
 - III. The first number is 3 or 12.
 - IV. The second number is 12.
 - V. The second number is 3.
 - VI. The second number is 3 or 12.
 - VII. The second number is 6.
 - VIII. The third number is 6.
 - IX. The third number is 3 or 6.
 - X. The third number is 12 or 3.
- a. (I), (IV) and (VIII)
 - b. (II), (IV) and (IX)
 - c. (II), (V) and (VIII)
 - d. (I), (VI) and (IX)
 - e. (III), (VII) and (X).

(2 points)

- 68.** The sum of the first four terms of a geometrical progression (G.P.) is 60 and the sum of the first six terms of the same G.P. is 252. The common ratio is 2.

What is the tenth term of the G.P. from the beginning?

- a. 64
- b. 256
- c. 1024
- d. 2048
- e. 4096.

(2 points)

69. Pioneer Electricals Company (PEC) produces and sells three types of parts, viz, **A**, **B** and **C**, required for manufacturing washing machines. These parts are sold to the manufacturers of washing machines. The firm purchases the casting of the parts from locally situated foundries and then finishes the three types of parts by drilling, shaping and polishing the same. The selling price of each unit, after finishing, of parts **A**, **B** and **C** are Rs.400, Rs.500 and Rs.700 respectively. All the finished parts can be sold. The castings for each unit of the parts **A**, **B** and **C**, cost Rs.250, Rs.300 and Rs.500 respectively. The firm possesses one drilling machine, one shaping machine and one finishing machine. The costs per hour for operating the three machines are Rs.500 for drilling, Rs.750 for shaping and Rs.400 for polishing. The drilling machine can process 50 units of part **A** per hour, or 25 units of part **B** per hour, or 25 units of part **C** per hour. The shaping machine can process 25 units of part **A** per hour, or 30 units of part **B** per hour, or 30 units of part **C** per hour. The polishing machine can process 50 units of part **A** per hour, or 40 units of part **B** per hour, or 20 units of part **C** per hour. The management of the firm has to decide how many units of each type of part it should produce per hour in order to maximize the profit for an hour of operation.

Let **A**, **B** and **C** denote the number of units of parts **A**, **B** and **C** respectively, produced per hour. Which of the following would constitute the **correct** representation of the above problem as a linear programming model?

- I. Objective function : Maximize profit (Z) = $135A + 145B + 102C$.
 - II. Objective function : Maximize profit (Z) = $135A + 102B + 145C$.
 - III. Objective function : Maximize profit (Z) = $145A + 135B + 102C$.
 - IV. Objective function : Maximize profit (Z) = $102A + 145B + 135C$.
 - V. Drilling machine constraint : $2A + B + C \leq 50$.
 - VI. Drilling machine constraint : $2A + 2B + C \leq 50$.
 - VII. Drilling machine constraint : $A + 2B + 2C \leq 50$.
 - VIII. Drilling machine constraint : $A + B + C \leq 50$.
 - IX. Shaping machine constraint : $5A + 5B + 5C \leq 150$.
 - X. Shaping machine constraint : $5A + 6B + 5C \leq 150$.
 - XI. Shaping machine constraint : $6A + 6B + 6C \leq 150$.
 - XII. Shaping machine constraint : $6A + 5B + 5C \leq 150$.
 - XIII. Polishing machine constraint : $5A + 4B + 10C \leq 200$.
 - XIV. Polishing machine constraint : $10A + 4B + 5C \leq 200$.
 - XV. Polishing machine constraint : $10A + 5B + 4C \leq 200$.
 - XVI. Polishing machine constraint : $4A + 5B + 10C \leq 200$.
- a. (I), (V), (IX) and (XIII)
 - b. (II), (VI), (X) and (XIV)
 - c. (III), (VIII), (XI) and (XV)
 - d. (IV), (VII), (XII) and (XVI)
 - e. (I), (VI), (XI) and (XIII).

(4 points)

70. The linear programming formulation of a problem is given below:

Maximize $Z = 80P + 60Q$

Subject to the constraints:

$$2P + Q \leq 80$$

$$P + Q \leq 60$$

$$P \leq 30$$

$$Q \leq 50$$

$$P, Q \geq 0$$

Solve the above problem graphically and find out which of the following statements are **correct**? (In the ordered pairs given below the value of **P** appears first and the value of **Q** appears next).

- I. The feasible region is bounded by the points (0,0), (0,60), (30,30) and (30,0).
 - II. The feasible region is bounded by the points (0,0), (0,80), (30,20) and (30,0).
 - III. The feasible region is bounded by the points (40,0), (30,20), (30,30) and (60,0).
 - IV. The feasible region is bounded by the points (0,0), (20,40), (30,20) and (30,0).
 - V. The feasible region is bounded by the points (0,0), (0,50), (10,50), (20,40), (30,20) and (30,0).
 - VI. The optimal combination is $P = 40$ and $Q = 20$.
 - VII. The optimal combination is $P = 40$ and $Q = 0$.
 - VIII. The optimal combination is $P = 60$ and $Q = 0$.
 - IX. The optimal combination is $P = 30$ and $Q = 20$.
 - X. The optimal combination is $P = 20$ and $Q = 40$.
 - XI. The optimal value of Z is 3200.
 - XII. The optimal value of Z is 4400.
 - XIII. The optimal value of Z is 4800.
 - XIV. The optimal value of Z is 2400.
 - XV. The optimal value of Z is 4000.
- a. (I), (VIII), and (XIII)
 - b. (II), (IX) and (XIV)
 - c. (III), (X) and (XV)
 - d. (IV), (XI), and (XVI)
 - e. (V), (X) and (XV).

(4 points)

END OF PART A

Suggested Answers

Quantitative Methods – I (July 2003)

Section A: Basic Concepts

1. Answer : (c)
Reason : The set of natural numbers:
Is an infinite set
Starts with 1
Does not include zero, fractions or negative numbers.
2. Answer : (d)
Reason : $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ is known as the associative property of multiplication.
3. Answer : (d)
Reason : According to the identity property of addition, the addition of zero to a real number returns back the number itself.
4. Answer : (e)
Reason : The least common multiple of a group of numbers is the smallest number that can be divided by each number of the group without leaving a remainder.
5. Answer : (c)
Reason : The set of rational numbers includes those numbers which are of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
6. Answer : (d)
Reason : If $a < b$ and $x < 0$ then $ax > bx$. All other statements are true.
7. Answer : (b)
Reason : $(a, b]$ is the interval which includes the member x such that $a < x \leq b$
 (a, b) includes the numbers x such that $a < x < b$
 $[a, b)$ includes the numbers x such that $a \leq x < b$
 $[a, b]$ includes the numbers x such that $a \leq x \leq b$.
8. Answer : (c)
Reason : A series of numbers is said to be in a geometrical progression if the successive numbers in the series increase or decrease by a common ratio.
9. Answer : (e)
Reason : $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $\therefore$ For $c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 0}}{2a} = \frac{-b \pm b}{2a} = 0$ or $-\frac{b}{a}$
10. Answer : (d)
Reason : The reciprocals of the terms in harmonic progression are in arithmetic progression.
11. Answer : (a)
Reason : If A , G and H are the arithmetic mean, geometric mean and harmonic mean between two given quantities, then
 $G = (AH)^{1/2}$
 $\therefore G$ is the geometric mean of A and H .

12. Answer : (b)

Reason : If a, b and c are in harmonic progression then, b is the harmonic mean of a and c.

$$b = \frac{2ac}{a+c}.$$

13. Answer : (a)

Reason : The sum of n terms of an A.P., $S_n = \frac{n}{2} \{2a + (n-1)d\}$

$$= \frac{n}{2} \{t_1 + t_n\}$$

$$S_n = 0 \quad \Rightarrow \quad \frac{n}{2} \{t_1 + t_n\} = 0$$

$$\Rightarrow t_1 + t_n = 0$$

$$\Rightarrow t_1 = -t_n.$$

14. Answer : (d)

Reason : ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!\{n-(n-r)\}} = {}^nC_{n-r}$

15. Answer : (e)

Reason : $\log_b a \times \log_a b = 1$.

16. Answer : (d)

Reason : $\log_a(M^n) = n \log_a M$

17. Answer : (c)

Reason : $\log_a(M \times N) = \log_a M + \log_a N$.

18. Answer : (e)

Reason : For any real number, the interval $(-\infty, a)$ represents the real numbers, x such that $-\infty < x < a$. This means, the set of all real numbers less than a.

19. Answer : (b)

Reason : The domain of a function, $y = f(x)$, includes the values of x for which the function is defined.

20. Answer : (d)

Reason : To every number in the domain, a function always associates a unique number in the range.

21. Answer : (d)

Reason : If the graph of a function involving one independent variable is a straight line, then the function is a linear function.

22. Answer : (d)

Reason : The constraint in equations specify the consumption of resources and the amount of resources available.

23. Answer : (c)

Reason : (a), (b), (d) and (e) are all assumptions underlying linear programming. However, (c) is not an assumption because linear programming assumes that the amount of resources consumed by each unit of the decision variables is certain.

24. Answer : (c)

Reason : If, for a function $y = f(x)$, $f'(x) = 0$, then this means that there is no change in $f(x)$ for any change in x. This means that $f(x)$ is constant for all values of x.

25. Answer : (d)

Reason : A function, $f(x)$, will have a relative maxima at $x = a$, if:

At $x = a$, $f'(x) = 0$ and $f''(x) < 0$.

26. Answer : (b)
Reason : If every item in a data set is increased by a constant C, then the arithmetic mean of the resulting data set will be equal to the mean of the original data set plus C, i.e., C + mean of the original data set.
27. Answer : (e)
Reason : The mean of the squared deviations of all the observations in a data set is equal to the variance of the data set.
28. Answer : (e)
Reason : If B, C and D are mutually exclusive and collectively exhaustive, and A is another event which can jointly occur with B, C or D, then

$$P(A) = P(A \text{ and } B) + P(A \text{ and } C) + P(A \text{ and } D).$$
This means that P(A) is equal to the sum of all the joint probabilities which include event A.
29. Answer : (a)
Reason : The exponential function falls from left to right as the values increase along the X-axis if $m > 0$ and $0 < a < 1$. Hence (a) is true. (b), (c) (d) and (e) are not true because they are not the characteristics of the exponential function.
30. Answer : (a)
Reason : In case of a negatively skewed distribution arithmetic mean < median < mode. Hence (a) is the answer. The relationships stated in (b), (c), (d) and (e) are not the features of negatively skewed distributions.
31. Answer : (d)
Reason : If the derivative is a function of the independent variable then the value of the derivative is dependent on the point where it is evaluated. All the other alternatives are true.
32. Answer : (a)
Reason : The totality of all possible outcomes of an experiment is called sample space.
33. Answer : (c)
Reason : Adjacent classes should not overlap. Hence (c) is not correct. Alternatives (a), (b) and (d) are correct.
34. Answer : (a)
Reason : If there are n items in data then the average deviation from zero can be written as
- $$\frac{(x_1 - 0) + (x_2 - 0) + \dots + (x_n - 0)}{n}$$
- $$= \frac{\sum_{i=1}^n x_i - 0 \times n}{n} = \frac{\sum_{i=1}^n x_i}{n} = \text{Mean}.$$
35. Answer : (c)
Reason : Baye's theorem helps the statistician to calculate posterior (or revised) probability.
36. Answer : (c)
Reason : The ogive is drawn using cumulative frequencies.
37. Answer : (b)
Reason : The sum of squared deviations of items from the arithmetic mean is less than the sum of the squared deviations of the items from any other value.
38. Answer : (d)
Reason : Co-efficient of variation is a relative measure of dispersion based upon standard deviation and mean.

$$\text{Co-efficient of variation} = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100\% .$$

39. Answer : (a)

Reason : Standard deviation is independent of change of origin i.e., it remains unchanged even if all the items in the data set are increased or decreased by the same quantity.

40. Answer : (c)

Reason : The appropriate mean for a set of ratios using the denominators of the ratios as weights is weighted arithmetic mean.

Section B: Problems

41. Answer : (d)

Reason : A function will be undefined at those values of the independent variables for which the function has a value $\frac{a}{0}$ (a is a real number) or $\frac{0}{0}$. Otherwise it is defined.

$$(a) \quad g(t) = \frac{t+5}{t^2+t-12} \quad \therefore g(3) = \frac{3+5}{3^2+3-12} = \frac{8}{0} \text{ (Undefined)}$$

$$(b) \quad g(t) = \frac{t+4}{t^2-3t-10} \quad \therefore g(-2) = \frac{-2+4}{(-2)^2-(3 \times -2)-10} = \frac{2}{4+6-10} = \frac{2}{0} \text{ (Undefined)}$$

$$(c) \quad g(t) = \frac{t-2}{t^2-9t+20} \quad \therefore g(3) = \frac{3-2}{3^2-(9 \times 3)+20} = \frac{1}{9-27+20} = \frac{1}{2} = \text{ (Defined)}$$

$$(d) \quad g(t) = \frac{t-1}{t^2-5t+6} \quad \therefore g(2) = \frac{2-1}{2^2-(5 \times 2)+6} = \frac{1}{4-10+6} = \frac{1}{0} \text{ (Undefined)}$$

$$(e) \quad g(t) = \frac{t-5}{t^2-6t+8} \quad \therefore g(2) = \frac{2-5}{2^2-(6 \times 2)+8} = \frac{-3}{4-12+8} = \frac{-3}{0} \text{ (Undefined)}$$

$$g(4) = \frac{4-5}{4^2-(6 \times 4)+8} = \frac{-1}{16-24+8} = \frac{-1}{0} \text{ (Undefined).}$$

42. Answer : (c)

Reason : A linear function, $f(x) = a + bx$

$$(a) \quad f(2) = 7, \quad f(7) = 32$$

$$\therefore \text{Slope, } b = \frac{f(7)-f(2)}{7-2} = \frac{32-7}{7-2} = \frac{25}{5} = 5.$$

$$(b) \quad g(-5) = 40, \quad g(2) = -9$$

$$\therefore \text{Slope, } b = \frac{g(2)-g(-5)}{2-(-5)} = \frac{-9-40}{7} = \frac{-49}{7} = -7.$$

Putting the value of b in $g(-5)$

$$\therefore g(-5) = 40 = a + (-7)(-5)$$

$$\text{or} \quad 40 = a + 35$$

$$\text{or} \quad a = 40 - 35 = 5$$

$$\therefore g(x) = 5 - 7x$$

$$\therefore g(0) = 5 - (7 \times 0) = 5$$

$$(c) \quad \frac{y}{5} - \frac{x}{2} = 5 \quad \text{or} \quad \frac{y}{5} = 5 + \frac{x}{2} \quad \text{or} \quad y = 25 + \frac{5}{2}x$$

$\therefore$ Intercept on Y axis is 25.

(d) When $x = 2$, $y = 20$. When $x = 5$, $y = 44$.

$$\therefore b = \frac{44-20}{5-2} = \frac{24}{3} = 8 \quad \therefore a = y - bx = 20 - (8 \times 2) = 4$$

$$\therefore y = 4 + 8x \quad \therefore \text{For } x = 8, y = 4 + (8 \times 8) = 68.$$

43. Answer : (c)

- Reason : (a) $g(x) = x^2 - 8x + 15 \quad \therefore g(-3) = (-3)^2 - (8 \times -3) + 15 = 9 + 24 + 15 = 48 > 0$
 (b) $g(x) = x^2 + 2x - 8 \quad \therefore g(4) = (4)^2 + (2 \times 4) - 8 = 16 + 8 - 8 = 16 > 0$
 (c) $g(x) = x^2 + x - 6 \quad \therefore g(-3) = (-3)^2 + (-3) - 6 = 0$
 (d) $g(x) = x^2 - x - 6 \quad \therefore g(2) = (2)^2 - (2) - 6 = -4 < 0$
 (e) $g(x) = x^2 - 9x + 20 \quad \therefore g(5) = 5^2 - (9 \times 5) + 20 = 0$

44. Answer : (b)

$$\text{Reason : (a) } \lim_{t \rightarrow 3} \frac{t^2 - 5t + 6}{t^2 - 9t + 18} = \lim_{t \rightarrow 3} \frac{(t-2)(t-3)}{(t-3)(t-6)} = \lim_{t \rightarrow 3} \frac{t-2}{t-6} = \frac{3-2}{3-6} = \frac{1}{-3} = -\frac{1}{3}$$

$$(b) \lim_{t \rightarrow 1} \frac{t^3 - 1}{t^2 - 1} = \lim_{t \rightarrow 1} \frac{(t-1)(t^2 + t + 1)}{(t-1)(t+1)} = \lim_{t \rightarrow 1} \frac{t^2 + t + 1}{t+1} = \frac{1^2 + 1 + 1}{1+1} = \frac{3}{2}$$

$$\begin{aligned} (c) \lim_{t \rightarrow 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t} &= \lim_{t \rightarrow 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t} \times \frac{\sqrt{1+t} + \sqrt{1-t}}{\sqrt{1+t} + \sqrt{1-t}} \\ &= \lim_{t \rightarrow 0} \frac{(1+t) - (1-t)}{t(\sqrt{1+t} + \sqrt{1-t})} \\ &= \lim_{t \rightarrow 0} \frac{1+t-1+t}{t(\sqrt{1+t} + \sqrt{1-t})} \\ &= \lim_{t \rightarrow 0} \frac{2t}{t(\sqrt{1+t} + \sqrt{1-t})} \\ &= \lim_{t \rightarrow 0} \frac{2}{\sqrt{1+t} + \sqrt{1-t}} = \frac{2}{\sqrt{1+0} + \sqrt{1-0}} = \frac{2}{2} = 1 \end{aligned}$$

$$(d) \lim_{t \rightarrow \infty} \frac{t+1}{t+2} = \lim_{t \rightarrow \infty} \frac{\frac{t+1}{t}}{\frac{t+2}{t}} = \lim_{t \rightarrow \infty} \frac{1 + \frac{1}{t}}{1 + \frac{2}{t}} = \frac{1+0}{1+0} = 1$$

45. Answer : (a)

$$\text{Reason : (a) } f(t) = \frac{\ln t}{t} \quad \therefore f'(t) = \frac{t \frac{d}{dt}(\ln t) - \ln t \frac{d}{dt}(t)}{t^2} = \frac{t \cdot \frac{1}{t} - \ln t \cdot 1}{t^2}$$

$$\text{or } f'(t) = \frac{1 - \ln t}{t^2}$$

$$(b) f(t) = (\ln t)^5 \quad \therefore f'(t) = 5 (\ln t)^{5-1} \cdot \frac{d}{dt}(\ln t) = 5 (\ln t)^4 \cdot \frac{1}{t}$$

$$\text{or } f'(t) = \frac{5}{t} (\ln t)^4$$

$$(c) f(t) = (2t+5)^5 \quad \therefore f'(t) = 5 (2t+5)^{5-1} \cdot \frac{d}{dt}(2t+5) = 5 (2t+5)^4 \cdot 2$$

$$\text{or } f'(t) = 10 (2t+5)^4$$

$$(d) \quad f(t) = e^{\sqrt{t}} \therefore f'(t) = e^{\sqrt{t}} \cdot \frac{d}{dt} \sqrt{t} = e^{\sqrt{t}} \left(\frac{1}{2} t^{-\frac{1}{2}} \right)$$

$$\text{or } f'(t) = \frac{e^{\sqrt{t}}}{2\sqrt{t}}$$

46. Answer : (c)

$$\text{Reason : } f(t) = 112t - \frac{28}{15}t^2 - 600$$

$$f'(t) = 112 - \frac{28}{15} \times 2t = 112 - \frac{56}{15}t$$

For maxima or minima $f'(t) = 0$

$$f'(t) = 0 \Rightarrow 112 - \frac{56}{15}t = 0 \text{ or } t = \frac{112 \times 15}{56} = 30$$

$$f''(t) = -\frac{56}{15} \text{ which is always negative}$$

Hence, $f(t)$ is maximum at $t = 30$.

47. Answer : (a)

$$\text{Reason : } f(x) = 15x^2 - 60x$$

$$f'(x) = 15(2x) - 60 = 30x - 60$$

For maxima or minima $f'(x) = 0$

$$f'(x) = 0 \Rightarrow 30x - 60 = 0 \text{ or } x = \frac{60}{30} = 2$$

$$f''(x) = 30 \text{ This is always positive}$$

$\therefore f(x)$ is minimum at $x = 2$.

48. Answer : (d)

$$\text{Reason : } f(x) = 3x^2 - (k+12)x + 200$$

$$f'(x) = 6x - (k+12)$$

$$f'(4) = 0 \Rightarrow 6(4) - (k+12) = 0 \text{ or } 24 = k+12 \text{ or } k = 24 - 12 = 12.$$

49. Answer : (d)

$$\text{Reason : Total cost is given by: } T(x) = 10500x - 20x^2 + \frac{1}{5}x^3$$

$$\text{Average cost} = \frac{\text{Total cost}}{\text{Number of units}}$$

$$\therefore \text{Average cost, } U(x) = \frac{1}{x} \left[10500x - 20x^2 + \frac{1}{5}x^3 \right]$$

$$\text{or } U(x) = 10500 - 20x + \frac{1}{5}x^2$$

$$U'(x) = -20 + \frac{2}{5}x$$

For maxima or minima $U'(x) = 0$

$$U'(x) = 0 \Rightarrow -20 + \frac{2}{5}x = 0 \text{ or } x = \frac{20 \times 5}{2} = 50$$

$$U''(x) = \frac{2}{5} > 0 \text{ always}$$

$\therefore$ The average cost of the product is minimum when number of units produced is 50.

50. Answer : (d)

Reason : Let the following notations be used:

M : Number of men

W : Number of women

N = M + W : Total number of men and women

$\bar{X}_M$: Mean age of the men = 40 years

$\bar{X}_W$: Mean age of the women = 30 years

$\bar{X}_{MW}$: Mean age of the men and women taken together = 36

$$\therefore \bar{X}_{MW} = \frac{M\bar{X}_M + W\bar{X}_W}{N}$$

Dividing the numerator and denominator by N we get

$$\bar{X}_{MW} = \frac{\frac{M}{N}\bar{X}_M + \frac{W}{N}\bar{X}_W}{\frac{N}{N}} = \frac{M}{N}\bar{X}_M + \frac{W}{N}\bar{X}_W$$

$$\frac{M}{N} = \text{Proportion of men in the group}$$

$$\frac{W}{N} = \text{Proportion of women in the group}$$

$$\text{Also, } \frac{M}{N} + \frac{W}{N} = 1 \text{ (Because } M + W = N\text{)}$$

$$\therefore \frac{W}{N} = 1 - \frac{M}{N}$$

$$\therefore \bar{X}_{MW} = \frac{M}{N}\bar{X}_M + \left(1 - \frac{M}{N}\right)\bar{X}_W = \frac{M}{N}\bar{X}_M + \bar{X}_W - \frac{M}{N}\bar{X}_W$$

$$\text{or } \bar{X}_{MW} = \frac{M}{N}(\bar{X}_M - \bar{X}_W) + \bar{X}_W$$

$$\text{or } \frac{M}{N} = \frac{(\bar{X}_{MW} - \bar{X}_W)}{(\bar{X}_M - \bar{X}_W)}$$

$$\therefore \text{Proportion of men} = \frac{M}{N} = \frac{36 - 30}{40 - 30} = 0.60$$

Hence percentage of men is $0.60 \times 100 = 60\%$

$$\therefore \text{Proportion of women} = 1 - \frac{M}{N} = 1 - 0.60 = 0.40$$

$$\therefore \text{Percentage of women} = 0.40 \times 100 = 40\%.$$

51. Answer : (e)

Reason : The appropriate average for quantities that change over time is the geometric mean.

$\therefore$ The average growth rate over the seven year period

= Geometric mean of the quantity ratios – 1

$$= [1.05 \times 1.08 \times 0.90 \times 0.96 \times 1.06 \times 1.10 \times 1.07]^{1/7} - 1$$

$$= 0.0291$$

i.e. 2.91%

52. Answer : (c)

Reason : Median = $\left[\frac{(N+1)/2 - (F+1)}{f_m} \right] W + L_m$

Class	Frequency (f)	Cumulative frequency
25 – 35	15	15
35 – 45	25	40
45 – 55	32	72
55 – 65	28	100
65 – 75	18	118
75 – 85	12	130
	130	

Position of the median = $\frac{N+1}{2} = 65.5\text{th item}$

From above can be seen that the median class is 45 – 55.

$$\begin{aligned} \therefore N &= 130 \\ L_m &= 45 \\ f_m &= 32 \\ F &= 40 \\ W &= 10 \end{aligned}$$

$$\therefore \text{Median wage} = \left[\frac{\left(\frac{131}{2} \right) - (40+1)}{32} \right] 10 + 45 = \text{Rs.} 52.66 \text{ (approx.)}$$

53. Answer : (a)

Reason : Sample standard deviation = $\sqrt{\frac{\sum x^2}{n-1} - \frac{n\bar{x}^2}{n-1}}$

$$\sum_{i=1}^8 x_i^2 = 2412$$

$$\begin{aligned} \sum_{i=1}^8 x_i &= \sum_{i=1}^4 x_i + \sum_{i=2}^5 x_i - \sum_{i=2}^4 x_i + \sum_{i=5}^8 x_i - x_5 & \text{Or} & \sum_{i=1}^8 x_i = \sum_{i=1}^4 x_i + \sum_{i=5}^8 x_i \\ &= 64 + 80 - 54 + 66 - 26 = 130 & & = 64 + 66 \\ & & & = 130 \end{aligned}$$

$$\therefore \bar{x} = \frac{\sum_{i=1}^8 x_i}{n} = \frac{130}{8} = 16.25$$

$$\begin{aligned} \therefore \text{Standard deviation of the sample} &= \sqrt{\frac{2412}{8-1} - \frac{8}{8-1} (16.25)^2} \\ &= 6.541. \end{aligned}$$

54. Answer : (c)

Reason : Harmonic mean of a set of 'n' quantities = $\frac{n}{\sum \frac{1}{x_i}}$

$$= \frac{8}{\frac{1}{20} + \frac{1}{15} + \frac{1}{30} + \frac{1}{22} + \frac{1}{12} + \frac{1}{25} + \frac{1}{18} + \frac{1}{16}}$$

$$= 18.313.$$

55. Answer : (d)

Reason : The combined geometric mean of the three sets of data after the expansion,

$$= \left[\left\{ (1.252)^4 \times 1.20 \right\} \times \left\{ (1.164)^8 \times 1.18 \right\} \times \left\{ (1.206)^5 \times 1.24 \right\} \right]^{1/20}$$

$$= (2.948483 \times 3.976545 \times 3.16343)^{1/20}$$

$$= 1.19802 \cong 1.198$$

56. Answer : (e)

Reason : Harmonic mean of a set of 'n' quantities, $H = \frac{n}{\sum \frac{1}{x_i}}$

$$\text{Given: } \sum \frac{1}{x_i} = \frac{7}{8} \quad H = 3\frac{3}{7} = \frac{24}{7}$$

$$\text{From above, } n = H \cdot \sum \frac{1}{x_i}$$

$$\therefore n = \frac{24}{7} \times \frac{7}{8} = 3$$

$$\text{Harmonic mean after the expansion} = \frac{n+2}{\frac{7}{8} + \frac{1}{5} + \frac{1}{10}} = \frac{3+2}{\frac{47}{40}}$$

$$= 4\frac{12}{47}$$

57. Answer : (e)

Reason : **Incorrect sample:**

$$\bar{x} = \frac{\sum x}{n} \quad \text{or} \quad n\bar{x} = \sum x = 50 \times 40 = 2000$$

$$\text{Variance, } s^2 = 22^2 = 484$$

$$s^2 = \frac{\sum x^2}{n-1} - \frac{n\bar{x}^2}{n-1}$$

$$\therefore 484 = \frac{\sum x^2}{49} - \frac{50 \times 40^2}{49}$$

$$\text{or } \sum x^2 = (484 \times 49) + (50 \times 40^2) = 103716$$

Correct sample:

$$\text{Sum of the observations, } \sum x = 2000 - 150 + 50 = 1900 \quad \therefore \bar{x} = \frac{\sum x}{n} = \frac{1900}{50} = 38$$

$$\therefore \text{Correct variance, } s^2 = \frac{\sum x^2}{n-1} - \frac{n\bar{x}^2}{n-1}$$

$$= \frac{83716}{49} - \frac{50 \times 38^2}{49} = 235.02$$

$$\therefore \text{Correct standard deviation} = \sqrt{s^2} = 15.33 \text{ (approx.)}$$

58. Answer : (c)

Reason : Let the following notations be used:

A : Getting a plumbing contract

B : Getting a wiring contract

$$\text{Given: } P(A) = \frac{2}{5}$$

$$P(B) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(A \text{ or } B) = \frac{17}{20}$$

Since the two types of contract are independent, we may consider them to be not mutually exclusive.

$$\therefore P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\text{or } P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$$

$$= \frac{2}{5} + \frac{3}{4} - \frac{17}{20} = \frac{3}{10} \text{ i.e. } 30\%.$$

59. Answer : (e)

Let the probability that the students P, Q, R, S and T will be able to solve the problem be denoted by P(P), P(Q), P(R), P(S), and P(T) respectively.

Given:

$$P(P) = \frac{2}{5} \quad P(Q) = \frac{2}{3} \quad P(R) = \frac{1}{6} \quad P(S) = \frac{1}{2} \quad P(T) = \frac{3}{8}$$

Probability that none of the students will be able to solve the problem

$$= \left(1 - \frac{2}{5}\right) \left(1 - \frac{2}{3}\right) \left(1 - \frac{1}{6}\right) \left(1 - \frac{1}{2}\right) \left(1 - \frac{3}{8}\right) = \frac{3}{5} \times \frac{1}{3} \times \frac{5}{6} \times \frac{1}{2} \times \frac{5}{8} = \frac{5}{96}$$

$$\therefore \text{Probability that the problem will be solved} = 1 - \frac{5}{96} = \frac{91}{96} = 0.948 \text{ i.e. } 94.8\%$$

60. Answer : (b)

Reason : Probability that the chosen item is either defective or it is nut

$$= P(\text{item is defective}) + P(\text{item is a nut}) - P(\text{item is a defective nut})$$

$$\text{Total number of items in the bin} = 125 + 200 = 325$$

$$\text{Total number of defective items} = \frac{1}{5} \times 125 + \frac{3}{4} \times 200 = 175 = 175$$

$$\text{Number of nuts} = 125$$

$$\text{Number of defective nuts} = \frac{1}{5} \times 125 = 25$$

$$P(\text{Item is defective or item is a nut}) = \frac{175}{325} + \frac{125}{325} - \frac{25}{325} = \frac{11}{13} = 0.846 \text{ i.e., } 84.6\%.$$

61. Answer : (e)

Reason : I. P(Part will be manufactured by B)

$$= P(\text{B and 0 defect}) + P(\text{B and 1 defect}) + P(\text{B and 2 defects})$$

$$= 0.24 + 0.10 + 0.06 = 0.40 \text{ i.e. } 40\%$$

II. P(Part will be manufactured by A)

$$= P(\text{A and 0 defect}) + P(\text{A and 1 defect}) + P(\text{A and 2 defects})$$

$$= 0.42 + 0.12 + 0.06 = 0.60 \text{ i.e. } 60\%$$

III. P(Part will have 1 defect) = P(1 defect and A) + P(1 defect and B) = 0.22 i.e. 22%.

IV. P(Part will have 2 defects) = P(2 defect and A) + P(2 defect and B) = 0.12 i.e. 12%

V. P(Part will have no defect) = P(0 defect and A) + P(0 defect and B) = 0.66 i.e 66%

$$\text{VI. } P(0 \text{ defect } / B) = \frac{P(0 \text{ defect and B})}{P(B)} = \frac{0.24}{0.40} = 0.60 \text{ i.e. } 60\%$$

$$\text{VII. } P(1 \text{ defect } / B) = \frac{P(1 \text{ defect and B})}{P(B)} = \frac{0.10}{0.40} = 0.25 \text{ i.e. } 25\%$$

$$\text{VIII. } P(2 \text{ defects } / B) = \frac{P(2 \text{ defects and B})}{P(B)} = \frac{0.06}{0.40} = 0.15 \text{ i.e. } 15\%$$

$$\text{IX. } P(2 \text{ defects } / A) = \frac{P(2 \text{ defects and A})}{P(A)} = \frac{0.06}{0.60} = 0.10 \text{ i.e. } 10\%$$

$$\text{X. } P(1 \text{ defect } / A) = \frac{P(1 \text{ defect and A})}{P(A)} = \frac{0.12}{0.60} = 0.20 \text{ i.e. } 20\%$$

$$\text{XI. } P(0 \text{ defect } / A) = \frac{P(0 \text{ defect and A})}{P(A)} = \frac{0.42}{0.60} = 0.70 \text{ i.e. } 70\%$$

62. Answer : (d)

Reason : Probability that A and B give contradictory statements

$$= P(\text{A tells truth and B tells lie}) + P(\text{A tells lie and B tells truth})$$

$$= \frac{4}{5} \times \left(1 - \frac{3}{4}\right) + \left(1 - \frac{4}{5}\right) \times \frac{3}{4}$$

$$= \frac{1}{5} + \frac{3}{20}$$

$$= \frac{7}{20} = 0.35 \text{ i.e. } 35\%$$

63. Answer : (c)

Reason : According to Baye's theorem:

$$P(\text{Female} / \text{Illiterate}) = \frac{P(\text{Female and Illiterate})}{P(\text{Illiterate})}$$

$$P(\text{Female and Illiterate}) = P(\text{Female}) \cdot P(\text{Illiterate} / \text{Female})$$

$$= \left(1 - \frac{3}{5}\right) \times \frac{4}{5} = \frac{2}{5} \times \frac{4}{5} = \frac{8}{25}$$

$$P(\text{Illiterate}) = P(\text{Female and Illiterate}) + P(\text{Male and Illiterate})$$

$$= \frac{8}{25} + [P(\text{Male}) \cdot P(\text{Illiterate} / \text{Male})]$$

$$= \frac{8}{25} + \left[\frac{3}{5} \times \frac{1}{2} \right] = \frac{8}{25} + \frac{3}{10} = \frac{31}{50}$$

$$\therefore P(\text{Female / Illiterate}) = \frac{\frac{8}{25}}{\frac{31}{50}} = \frac{16}{31} = 0.516 \text{ i.e. } 51.6\% \text{ approximately.}$$

(In the above calculations, 'males' imply adult males and 'females' imply adult females).

64. Answer : (a)

Reason : Let 'a' be the first term and 'd' be the common difference in the A.P.

$$\therefore \text{ jth term, } t_j = a + (j - 1) d$$

$$\text{ kth term, } t_k = a + (k - 1) d$$

$$\text{Given: } j [a + (j - 1)d] = k [a + (k - 1) d]$$

$$\text{or } ja + j(j - 1)d = ka + k(k - 1) d$$

$$\text{or } a(j - k) = [k(k - 1) - j(j - 1)] d$$

$$\text{or } a(j - k) = [k^2 - k - j^2 + j] d$$

$$\text{or } a(j - k) = [k^2 - j^2 - (k - j)] d$$

$$\text{or } a(j - k) = [(k - j)(k + j) - (k - j)] d$$

$$\text{or } a(j - k) = [(k - j)(k + j - 1)] d$$

$$\text{or } a = \frac{(k - j)}{(j - k)}(k + j - 1)d$$

$$\text{or } a = -(j + k - 1) d \quad \dots\dots\dots (1)$$

$$(j + k)^{\text{th}} \text{ term, } t_{(j+k)} = a + (j + k - 1) d$$

From (1) above,

$$t_{(j+k)} = -(j + k - 1) d + (j + k - 1) d = 0$$

65. Answer : (d)

Reason : Given: $t_1 = a = 4$

Last term, $t_n = 24$

Sum of the terms, $S_n = 154$

In an A.P. sum of 'n' terms starting from the first term,

$$S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} (a + t_n)$$

$\therefore$ From above

$$154 = \frac{n}{2} [4 + 24] = 14 n$$

$$\text{or } n = \frac{154}{14} = 11$$

$$t_n = a + (n - 1) d$$

$$\therefore t_{11} = 4 + (11 - 1) d = 24$$

$$\text{or } 4 + 10 d = 24$$

$$\text{or } d = \frac{24 - 4}{10} = 2$$

66. Answer : (e)

Reason : Let the three numbers be $(a - d)$, a and $(a + d)$

$$\text{Given: } a - d + a + a + d = 18 \Rightarrow 3a = 18 \text{ or } a = \frac{18}{3} = 6$$

$$(a - d)^2 + a^2 + (a + d)^2 = 140$$

$$\text{or } a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 140$$

$$\text{or } 3a^2 + 2d^2 = 140$$

$$\text{or } 3(6^2) + 2d^2 = 140$$

$$\text{or } 108 + 2d^2 = 140$$

$$\text{or } d^2 = \frac{32}{2} = 16$$

$$\therefore d = \sqrt{16} = \pm 4$$

$\therefore$ The numbers are: 2, 6 and 10 or
10, 6 and 2

67. Answer : (e)

Reason : Let the numbers be $\frac{a}{r}$, a , ar

$$\text{Given: } \frac{a}{r} \times a \times ar = 216 \Rightarrow a^3 = 216 \Rightarrow a = 6$$

$$\frac{a}{r} + a + ar = 21$$

$$\text{or } a \left(\frac{1}{r} + 1 + r \right) = 21$$

$$\text{or } 6 \left(\frac{r^2 + r + 1}{r} \right) = 21$$

$$\text{or } 6(r^2 + r + 1) = 21r$$

$$\text{or } 6r^2 + 6r + 6 = 21r$$

$$\text{or } 6r^2 - 15r + 6 = 0$$

$$\text{or } 2r^2 - 5r + 2 = 0$$

$$\text{or } 2r^2 - (4 + 1)r + 2 = 0$$

$$\text{or } 2r^2 - 4r - r + 2 = 0$$

$$\text{or } 2r(r - 2) - (r - 2) = 0$$

$$\text{or } (r - 2)(2r - 1) = 0$$

$$\Rightarrow r = 2 \quad \text{or} \quad r = \frac{1}{2}$$

$\therefore$ The numbers are:

$$\text{For } r = 2 \text{ and } a = 6 \quad \frac{a}{r} = \frac{6}{2} = 3 \quad a = 6 \quad ar = 6 \times 2 = 12$$

i.e. 3, 6 and 12

$$\text{For } r = \frac{1}{2} \text{ and } a = 6 \quad \frac{a}{r} = \frac{6}{\left(\frac{1}{2}\right)} = 12 \quad a = 6 \quad ar = 6 \times \frac{1}{2} = 3$$

i.e. 12, 6 and 3.

68. Answer : (d)

Reason : Sum of the first four terms, $S_4 = 60$

Sum of the first six terms, $S_6 = 252$

$$S_6 - S_4 = 252 - 60$$

$$\text{or } S_6 - S_5 + S_5 - S_4 = 192 \text{ (where } S_5 \text{ is the sum of first five terms)}$$

$$\text{or } (S_6 - S_5) + (S_5 - S_4) = 192 \dots \dots (A)$$

Now, $S_6 = S_5 + t_6$ (where t_6 is the sixth term)

$$\Rightarrow S_6 - S_5 = t_6$$

$$S_5 = S_4 + t_5 \text{ (where } t_5 \text{ is the fifth term)}$$

$$\Rightarrow S_5 - S_4 = t_5$$

$\therefore$ Equation (A) may be rewritten as -----

$$t_6 + t_5 = 192$$

Now, $t_6 = t_5 \times r$ (since the terms are in G.P. each term will be equal to the product of its preceding term and common ratio, r).

$$\therefore t_5 \cdot r + t_5 = 192$$

$$\text{or } t_5 (r + 1) = 192$$

$$\text{or } t_5 = \frac{192}{r+1}$$

$$\therefore t_5 = \frac{192}{2+1} = 64 \text{ (} r = 2, \text{ given)}$$

$$t_5 = ar^{5-1} = ar^4 \text{ (where 'a' is the first term)}$$

$$\therefore a(2^4) = 64 \text{ or } a = \frac{64}{2^4} = 4$$

$$\therefore t_{10} = ar^{10-1} = ar^9 = 4(2^9) = 2048$$

69. Answer : (d)

$$\text{Reason : Profit per unit of part A} = (400 - 250) - \left(\frac{500}{50} + \frac{750}{25} + \frac{400}{50} \right) = \text{Rs.102}$$

$$\text{Profit per unit of part B} = (500 - 300) - \left(\frac{500}{25} + \frac{750}{30} + \frac{400}{40} \right) = \text{Rs.145}$$

$$\text{Profit per unit of part C} = (700 - 500) - \left(\frac{500}{25} + \frac{750}{30} + \frac{400}{20} \right) = \text{Rs.135}$$

Let the following notations be used:

A : No. of units of part A produced per hour

B : No. of units of part B produced per hour

C : No. of units of part C produced per hour

Time consumed per unit (in hour) of each part:

Part	A	B	C
Operation:			
Drilling	1/50	1/25	1/25
Shaping	1/25	1/30	1/30
Polishing	1/50	1/40	1/20

The LPP formulation is:

$$\text{Maximize : Profit (Z) = } 102A + 145B + 135C$$

$$\begin{aligned}
 \text{Subject to : } & \frac{1}{50} A + \frac{1}{25} B + \frac{1}{25} C \leq 1 && \text{(Drilling machine constraint)} \\
 \text{or} & A + 2B + 2C \leq 50 && \text{(Multiplying 50 both sides)} \\
 & \frac{1}{25} A + \frac{1}{30} B + \frac{1}{30} C \leq 1 && \text{(Shaping machine constraint)} \\
 \text{or} & 6A + 5B + 5C \leq 150 && \text{(Multiplying 150 both sides)} \\
 & \frac{1}{50} A + \frac{1}{40} B + \frac{1}{20} C \leq 1 && \text{(Polishing machine constraint)} \\
 \text{or} & 4A + 5B + 10C \leq 200 && \text{(Multiplying 200 both sides)}
 \end{aligned}$$

70. Answer : (e)

Reason : It can be observed from the graph that the feasible region is bounded by the points: (0, 0), (0, 50), (10, 50), (20, 40), (30, 20) and (30, 0).

The horizontal axis represents P and vertical axis represents Q

Evaluating the objective function at the above corner points:

Point	Profit (Z) = 80 P + 60 Q
(0, 0)	Z = 80 (0) + 60 (0) = 0
(0, 50)	Z = 80 (0) + 60 (50) = 3000
(10, 50)	Z = 80 (10) + 60 (50) = 3800
(20, 40)	Z = 80 (20) + 60 (40) = 4000
(30, 20)	Z = 80 (30) + 60 (20) = 3600
(30, 0)	Z = 80 (30) + 60 (0) = 2400

The maximum value of the objective function is 4000 at corner point (20, 40)

∴ The optimal combination is:

$$P = 20$$

$$Q = 40.$$

